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N=31, D=11*

Ulf Gran
Institute for Theoretical Physics, K.U. Leuven,
Celestijnenlaan 200D, B-3001 Leuven, Belgium
E-mail: ulf.gran@fys.kuleuven.be

Jan Gutowski
DAMTP, Centre for Mathematical Sciences, University of Cambridge,
Wilberforce Road, Cambridge, CB3 0WA, U.K.
E-mail: J.B.Gutowski@damtp.cam.ac.uk

George Papadopoulos
Department of Mathematics, King's College London,
Strand, London WC2R 2LS, U.K.
E-mail: george.papadopoulos@kcl.ac.uk

Diederik Roest
Departament Estructura i Constituents de la Materia
Facultat de Física, Universitat de Barcelona
Diagonal, 647, 08028 Barcelona, Spain
E-mail: droest@ecm.ub.es

Abstract: We show that eleven-dimensional supergravity backgrounds with thirty one supersymmetries, $N = 31$, admit an additional Killing spinor and so they are locally isometric to maximally supersymmetric ones. This rules out the existence of simply connected eleven-dimensional supergravity preons. We also show that $N = 15$ solutions of type I supergravities are locally isometric to Minkowski spacetime.

Keywords: M-Theory, Superstring Vacua, Extended Supersymmetry, Supergravity Models.

*The laconic title has been inspired by that of [1].

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Contents

1. Introduction

The spinorial geometry technique is an effective tool to solve the Killing spinor equations of supergravity theories [2]. It is based on the use of gauge symmetry of the Killing spinor equations, on a description of spinors in terms of forms and on an oscillator basis in the space of spinors. Recently, it has been adapted to investigate backgrounds with near maximal number of supersymmetries. In particular it was found that IIB supergravity backgrounds with 31 supersymmetries, $N = 31$, are maximally supersymmetric [3]. This was extended in [4], using a different method, to show that IIA $N = 31$ supergravity backgrounds are also maximally supersymmetric. Later the spinorial geometry approach was applied to lower-dimensional supergravities[1] with similar results [5].

In this paper, we shall show that the $N = 31$ backgrounds of eleven-dimensional supergravity [6], termed as preons in [7], admit locally an additional Killing spinor and so they are maximally supersymmetric. Although this result is similar to those in type II supergravities mentioned above, there are some differences. To establish the type II results, the algebraic Killing spinor equations of type II supergravities have been instrumental. The remaining parallel transport equations were not explicitly solved and instead the result

[1]Technical innovations developed for this paper have been applied to establish the results in lower dimensions. This project precedes those in [5].

followed by an indirect argument. The eleven-dimensional supergravity does not have an algebraic Killing spinor equation. So to show that the $N = 31$ backgrounds are locally isometric to the maximally supersymmetric ones, the parallel transport equation

$$\mathcal{D}_A \epsilon^r = 0\,, \quad r = 1, \ldots, 31\,, \tag{1.1}$$

has to be solved explicitly. For this, one investigates the first integrability condition

$$\mathcal{R}_{AB} \epsilon^r = [\mathcal{D}_A, \mathcal{D}_B] \epsilon^r = 0\,, \tag{1.2}$$

where $\mathcal{R}$ is the supercovariant curvature. The stability subgroup, $\mathrm{Stab}(\epsilon)$, of 31 spinors ϵ in the holonomy group $\mathrm{SL}(32, \mathbb{R})$ is $\mathrm{Stab}(\epsilon) = \mathbb{R}^{31}$ [8 – 10]. Thus the integrability condition leaves undetermined 31 components of $\mathcal{R}$ represented by 31 two-forms u^r_{AB}. The task is to show that these components vanish as well and so the (reduced) holonomy of the supercovariant connection for 31 Killing spinors is in fact $\{1\}$. To do this, we shall use a modification of the procedure outlined in [3] which utilizes the normal ν of the Killing spinors ϵ^r and which is explained in the next section. Then we shall use the Bianchi identities, the field equations and the explicit expression of $\mathcal{R}$ in terms of the fields of eleven-dimensional supergravity to show that the supercovariant curvature vanishes, $\mathcal{R} = 0$. The latter condition is sufficient to demonstrate that the $N = 31$ backgrounds are locally isometric to the maximally supersymmetric ones. The maximally supersymmetric backgrounds have been classified in [11], and has been shown to be locally isometric to Minkowski space $\mathbb{R}^{10,1}$, the Freund-Rubin [12] spaces $AdS_4 \times S^7$ and $AdS_7 \times S^4$, and the Kowalski-Glikman plane wave [13], see also [14].

On non-simply connected spacetimes, the vanishing of the supercovariant curvature, $\mathcal{R} = 0$, does not always imply the existence of 32 linearly independent solutions for the parallel transport equation (1.1). There is also the additional subtlety of the existence of different spin structures on non-simply connected spacetimes. Since we show that the $N = 31$ backgrounds are locally isometric to the maximally supersymmetric ones, one may be able to construct $N = 31$ supersymmetric backgrounds by identifying one of the maximal supersymmetric ones with a discrete subgroup of its symmetry group. A large class of such backgrounds were constructed in [15] after identification with a cyclic subgroup of the symmetry group. None of these preserve 31 supersymmetries.[2] This may indicate that non-simply connected backgrounds with $N = 31$ supersymmetries do not exist but some further investigation is required to establish this. The absence of $N = 31$ supersymmetric backgrounds will be in agreement with a conjecture in [16] which was formulated under the assumption that the Killing spinors must lie in certain representations of subgroups of $\mathrm{Spin}(10, 1)$.

We also show that the $N = 15$ solutions in type I supergravities are locally maximally supersymmetric. This easily follows from our result in IIB [3] and the investigation of the Killing spinor equations of the heterotic supergravity in [17].

The paper has been organized as follows. In section two, we explain the procedure we use to investigate backgrounds with 31 supersymmetries and collect some useful formulae. In addition, we show that there are two cases to examine depending on the stability

[2] We thank J. Figueroa O'Farrill for helpful discussions on this point.

subgroup of the normal ν to the 31 Killing spinors in Spin(10, 1). In section three, we investigate the $N = 31$ backgrounds whose normal ν has stability subgroup SU(5), and in section four we examine the $N = 31$ backgrounds whose normal ν has stability subgroup $(\mathrm{Spin}(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$. In both cases, we establish that the $N = 31$ backgrounds are locally isometric to the maximally supersymmetric ones. In section five, we examine the $N = 15$ backgrounds of type I supergravities. In section six, we present our conclusions.

2. Supercurvature and Killing spinors

As we have mentioned, a consequence of the Killing spinor equations is the integrability condition (1.2). In [3], it was proposed to solve this condition directly. This has been facilitated by first using the gauge symmetry of the Killing spinor equations to choose the direction of the normal spinor ν of the $N = 31$ Killing spinors. In turn the gauge symmetry orients the hyperplane of the 31 Killing spinors along particular directions. This simplifies the expression for the Killing spinors and then using spinorial geometry the condition $\mathcal{R}\epsilon^r = 0$ gives rise to a linear system for the various components of the supercurvature. The linear system can be solved to give the conditions on $\mathcal{R}$ imposed by supersymmetry. Although this is the original way that we have tackled the problem, it turns out there is a simpler way to explore the integrability condition (1.2). For this let ϵ^r, $r = 1, \ldots, N$, be a basis in the space of Killing spinors and extend it as $(\epsilon^r, \tilde{\epsilon}^q)$, $q = N+1, \ldots, 32$ to a basis in the space of spinors. Then observe that the supercovariant curvature for a background with N Killing spinors can be written as

$$\mathcal{R}_{MN,ab} = U_{MN,rp}\, \epsilon^r_a \nu^p_b + U_{MN,pq}\, \tilde{\epsilon}^p_a \nu^q_b\,, \tag{2.1}$$

where ν^p are normal to the Killing spinors, $a, b = 1, \ldots, 32$ are spinor indices,

$$B(\epsilon^r, \nu^q) = 0\,, \tag{2.2}$$

and U are spacetime dependent two-forms. (Throughout this paper we use the conventions of [18].) Clearly (2.1) satisfies the integrability condition (1.2) because of (2.2). Since the holonomy of the supercovariant connection is contained in $\mathrm{SL}(32, \mathbb{R})$, one finds that

$$U_{MN,pq}\, B(\tilde{\epsilon}^p, \nu^q) = 0\ . \tag{2.3}$$

Taking into account this condition, the number of independent two-forms U that appear in (2.1) is $32^2 - 32N - 1$ which is the dimension of the stability subgroup $\mathrm{SL}(32 - N, \mathbb{R}) \ltimes (\oplus_N \mathbb{R}^{32-N})$ of N spinors in $\mathrm{SL}(32, \mathbb{R})$, see [8–10, 19].

In many cases of interest, the Killing spinors can be (locally) expressed in terms of a convenient basis η^r as

$$\epsilon^r = f^r{}_s \eta^s\,, \tag{2.4}$$

where f is an $N \times N$ invertible matrix of spacetime functions. If $(\eta^r, \tilde{\eta}^p)$ is a basis in the space of spinors, then (2.1) can be written as

$$\mathcal{R}_{MN,ab} = u_{MN,rp}\, \eta^r_a \nu^p_b + u_{MN,pq}\, \tilde{\eta}^p_a \nu^q_b\,, \tag{2.5}$$

where U and u are related in a straightforward way.

The supercurvature can be written as

$$\mathcal{R}_{MN,ab} = \sum_{k=1}^{5} \frac{1}{k!} (T^k_{MN})_{A_1 A_2 \ldots A_k} (\Gamma^{A_1 A_2 \ldots A_k})_{ab} \,, \tag{2.6}$$

where T^k depends on the frame e and four-form field strength F of eleven-dimensional supergravity. The relevant expressions[3] can be found in [20, 11]. It is also known that

$$\eta_a \theta_b = \frac{1}{32} \sum_{k=0}^{5} \frac{(-1)^{k+1}}{k!} B(\eta, \Gamma_{A_1 A_2 \ldots A_k} \theta) \, (\Gamma^{A_1 A_2 \ldots A_k})_{ab} \ . \tag{2.7}$$

This in particular implies that

$$(T^k_{MN})_{A_1 A_2 \ldots A_k} = \frac{(-1)^{k+1}}{32} [u_{MN,ip} \, B(\eta^i, \Gamma_{A_1 A_2 \ldots A_k} \nu^p) + u_{MN,pq} \, B(\tilde{\eta}^p, \Gamma_{A_1 A_2 \ldots A_k} \nu^q)] \tag{2.8}$$

subject to the condition (2.3) which can now be rewritten as

$$u_{MN,pq} \, B(\tilde{\eta}^p, \nu^q) = 0 \ . \tag{2.9}$$

The conditions (2.8) are equivalent to those that arise from the direct solution of the integrability condition (1.2). The advantage is that (2.8) is more easy to handle.

The conditions (2.8) and (2.9) can be easily adapted to backgrounds with 31 supersymmetries to find

$$(T^k_{MN})_{A_1 A_2 \ldots A_k} = \frac{(-1)^{k+1}}{32} u_{MN,r} \, B(\eta^r, \Gamma_{A_1 A_2 \ldots A_k} \nu) \ . \tag{2.10}$$

The second term in the r.h.s of (2.8) vanishes because of (2.9). This formula is consistent with the requirement that the holonomy of the supercovariant connection for $N = 31$ configurations is in $\mathbb{R}^{31}$.

Apart from the restrictions required by holonomy and described above, the supercovariant curvature $\mathcal{R}$ satisfies additional conditions which arise from the field equations, the Bianchi identities of the Riemann curvature R of the spacetime and of the four-form field strength F of eleven-dimensional supergravity, and the explicit expression of the components of $\mathcal{R}$ in terms of the fields. We can derive some of them by observing that $\Gamma^N \mathcal{R}_{MN}$ is a linear combination of field equations and Bianchi identities, and so it necessarily vanishes. In turn this leads to the vanishing of the following linear combinations of the components of $\mathcal{R}$:

$$\begin{aligned}
&(T^1_{MN})^N = 0 \,, \qquad (T^2_{MN})_P{}^N = 0 \,, \qquad (T^1_{MP_1})_{P_2} + \tfrac{1}{2} (T^3_{MN})_{P_1 P_2}{}^N = 0 \,, \\
&(T^2_{M[P_1})_{P_2 P_3]} - \tfrac{1}{3} (T^4_{MN})_{P_1 P_2 P_3}{}^N = 0 \,, \quad (T^3_{M[P_1})_{P_2 P_3 P_4]} + \tfrac{1}{4} (T^5_{MN})_{P_1 \cdots P_4}{}^N = 0 \,, \\
&(T^4_{M[P_1})_{P_2 \cdots P_5]} - \frac{1}{5 \cdot 5!} \epsilon_{P_1 \cdots P_5}{}^{Q_1 \cdots Q_6} (T^5_{MQ_1})_{Q_2 \cdots Q_6} = 0 \ .
\end{aligned} \tag{2.11}$$

[3]There are some apparent typos in the expression for $\mathcal{R}$ in [11].

The second and third of these equations are consequences of the Einstein and F field equations, respectively. We shall also use the additional conditions

$$(T^1_{MN})_P = (T^1_{[MN})_{P]}\,, \qquad (T^2_{MN})_{PQ} = (T^2_{PQ})_{MN}\,, \qquad (T^3_{[MN})_{PQR]} = 0\,, \quad (2.12)$$

which can be easily derived by inspecting the explicit expressions of T^k in terms of the physical fields in [11] and by using the Bianchi identity of F. Observe that the first condition in (2.11) is a consequence of the first condition in (2.12). It will turn out that (2.11), (2.12), the expression of T^k in terms of the physical fields and the conditions (2.10) are sufficient for the proof that we shall present.

It has been known for some time that there are two kinds of orbits of Spin(10, 1) in the space of Majorana spinors of eleven-dimensional supergravity. One has stability subgroup SU(5) and the other has stability subgroup $(\mathrm{Spin}(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$ [21, 22]. Therefore, there are two cases of $N = 31$ backgrounds to explore depending on in which orbit the normal ν of the Killing spinors lies. This is similar to the $N = 31$ IIB backgrounds in [3]. The Killing spinor equations for the associated $N = 1$ eleven-dimensional backgrounds have been solved in [23]. We shall investigate the two $N = 31$ cases separately.

3. SU(5)-invariant normal

3.1 Integrability conditions

To derive the conditions that the integrability of the Killing spinor equations imposes on the supercurvature, without loss of generality, we choose the normal of the 31 Killing spinors as

$$\nu = 1 + e_{12345}\,, \quad (3.1)$$

in the "time-like" spinor basis of [18]. Then the Killing spinors can be written as

$$\epsilon^r = f^r{}_s \eta^s\,, \quad (3.2)$$

where f is a 31×31 invertible matrix of real spacetime functions and η^s is a basis of 31 linearly independent Majorana spinors. This basis can be chosen as

$$\begin{aligned}
&\eta^0 = \nu = 1 + e_{12345}\,,\\
&\eta^k = -i(e_k - \frac{1}{4!}\epsilon_k{}^{q_1q_2q_3q_4} e_{q_1q_2q_3q_4})\,, \quad \eta^{5+k} = e_k + \frac{1}{4!}\epsilon_k{}^{q_1q_2q_3q_4} e_{q_1q_2q_3q_4}\,,\\
&\eta^{kl} = e_{kl} - \frac{1}{3!}\epsilon_{kl}{}^{q_1q_2q_3} e_{q_1q_2q_3}\,, \quad \hat\eta^{kl} = i(e_{kl} + \frac{1}{3!}\epsilon_{kl}{}^{q_1q_2q_3} e_{q_1q_2q_3})\,,
\end{aligned} \quad (3.3)$$

where $k, l = 1, \ldots, 5$. Observe that the linearly independent Majorana spinor $i - ie_{12345}$ is not orthogonal to the normal ν and so it has been excluded from the basis. It is convenient for what follows to set $\theta^0 = \eta^0$ and then choose a 'holomorphic' basis for the rest of the spinors as

$$\begin{aligned}
&\theta^\alpha = \eta^\alpha + i\eta^{\alpha+5}\,, \quad \theta^{\bar\alpha} = \eta^\alpha - i\eta^{\alpha+5}\,,\\
&\theta^{\alpha\beta} = \eta^{\alpha\beta} + i\hat\eta^{\alpha\beta}\,, \quad \theta^{\bar\alpha\bar\beta} = \eta^{\alpha\beta} - i\hat\eta^{\alpha\beta}\,,
\end{aligned} \quad (3.4)$$

i.e. decompose $\mathbf{31} = \mathbf{1}\oplus\mathbf{5}\oplus\bar{\mathbf{5}}\oplus\mathbf{10}\oplus\bar{\mathbf{10}}$ in SU(5) representations and so $r = (0, \alpha, \bar{\alpha}, \alpha\beta, \bar{\alpha}\bar{\beta})$. This has the advantage that the conditions (2.10) can be expressed in an SU(5) covariant manner.

To find the conditions that arise from the integrability condition (2.10), it is necessary to compute the spinor bi-linear forms. These have been presented in appendix A. It is then straightforward to see that (2.10) implies that

$$u_{MN,0} = -(T^1_{MN})_0\,, \quad u_{MN,\alpha} = (T^1_{MN})_\alpha\,, \quad u_{MN,\alpha\beta} = \frac{1}{2\sqrt{2}} i (T^2_{MN})_{\alpha\beta}\,. \tag{3.5}$$

In addition, (2.10) gives

$$\begin{aligned}
(T^2)_{0\alpha} &= i(T^1)_\alpha\,,\\
(T^2)_{\alpha\bar{\beta}} &= -i g_{\alpha\bar{\beta}}(T^1)_0\,,\\
(T^3)_{0\beta_1\beta_2} &= -i(T^2)_{\beta_1\beta_2}\,,\\
(T^3)_{0\alpha\bar{\beta}} &= 0\,,\\
(T^3)_{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} &= \tfrac{1}{2}\sqrt{2}\epsilon_{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3}{}^{\alpha_1\alpha_2}(T^2)_{\alpha_1\alpha_2}\,,\\
(T^3)_{\alpha\bar{\beta}_1\bar{\beta}_2} &= -2(T^1)_{[\bar{\beta}_1} g_{\bar{\beta}_2]\alpha}\,,\\
(T^4)_{0\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} &= -\tfrac{1}{2}\sqrt{2}i\epsilon_{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3}{}^{\alpha_1\alpha_2}(T^2)_{\alpha_1\alpha_2}\,,\\
(T^4)_{0\alpha\bar{\beta}_1\bar{\beta}_2} &= 2i(T^1)_{[\bar{\beta}_1} g_{\bar{\beta}_2]\alpha}\,,\\
(T^4)_{\alpha_1\alpha_2\alpha_3\alpha_4} &= -2\sqrt{2}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}{}^{\bar{\beta}}(T^1)_{\bar{\beta}}\,,\\
(T^4)_{\alpha\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} &= 3(T^2)_{[\bar{\beta}_1\bar{\beta}_2} g_{\bar{\beta}_3]\alpha}\,,\\
(T^4)_{\alpha_1\alpha_2\bar{\gamma}_1\bar{\gamma}_2} &= 0\,,\\
(T^5)_{0\alpha_1\alpha_2\alpha_3\alpha_4} &= 2\sqrt{2}i\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}{}^{\bar{\beta}}(T^1)_{\bar{\beta}}\,,\\
(T^5)_{0\alpha\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} &= 3i(T^2)_{[\bar{\beta}_1\bar{\beta}_2} g_{\bar{\beta}_3]\alpha}\,,\\
(T^5)_{0\alpha_1\alpha_2\bar{\beta}_1\bar{\beta}_2} &= -2(T^1)_0 g_{\alpha_1[\bar{\beta}_1} g_{\bar{\beta}_2]\alpha_2}\,,\\
(T^5)_{\alpha_1\cdots\alpha_5} &= 2\sqrt{2}i\epsilon_{\alpha_1\cdots\alpha_5}(T^1)_0\,,\\
(T^5)_{\alpha\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3\bar{\beta}_4} &= -\sqrt{2}\epsilon_{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3\bar{\beta}_4}{}^{\gamma}(T^2)_{\alpha\gamma}\,,\\
(T^5)_{\alpha_1\alpha_2\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} &= -6(T^1)_{[\bar{\beta}_1} g_{|\alpha_1|\bar{\beta}_2} g_{\bar{\beta}_3]\alpha_2}\,,
\end{aligned} \tag{3.6}$$

where we have suppressed the two-form indices. Observe that all T^k have been expressed in terms of T^1 and T^2. The above conditions are equivalent to the integrability condition $\mathcal{R}\epsilon^r = 0$. Clearly, they do not imply that $\mathcal{R} = 0$. It now remains to impose the conditions (2.11) and (2.12).

3.2 Solving the conditions

We shall first show using (2.11), (2.12) and (3.6) that T^1 vanishes. For this observe that (3.6) together with the skew-symmetry of T^1, (2.12), implies that

$$(T^2_{0\alpha})_{\beta\bar{\gamma}} = -iT^1_{0\alpha 0}\, g_{\beta\bar{\gamma}} = 0\,. \tag{3.7}$$

Using the symmetry property of T^2 in (2.12), this leads to

$$(T^2_{\beta\bar{\gamma}})_{0\alpha} = iT^1_{\alpha\beta\bar{\gamma}} = 0\,, \tag{3.8}$$

and hence the $(2,1)$ and $(1,2)$ parts of T^1 vanish. Turning to the field equations, using the Einstein equation in (2.11) and (3.6), we find that

$$T^1_{0\alpha}{}^{\alpha} = T^1_{0\alpha\beta} = 0\,, \qquad (T^2_{\alpha\gamma})_{\bar\beta}{}^{\gamma} = -2iT^1_{0\alpha\bar\beta}\,. \tag{3.9}$$

Similarly, the gauge field equation in (2.11) leads to

$$T^1_{\alpha\beta\gamma} = 0\,. \tag{3.10}$$

The only remaining component of T^1 is the traceless part of $T^1_{0\alpha\bar\beta}$. Its relation to T^2 is

$$(T^2_{\alpha\bar\beta})_{\gamma\bar\delta} = -iT^1_{0\alpha\bar\beta}\, g_{\gamma\bar\delta}\,. \tag{3.11}$$

Tracing this expression with $g^{\gamma\bar\delta}$ and using the symmetry in the two pairs of indices of T^2, this gives

$$(T^2_{\alpha\bar\beta})_{\gamma}{}^{\gamma} = -5iT^1_{0\alpha\bar\beta} = -ig_{\alpha\bar\beta}T^1_{0\gamma}{}^{\gamma} = 0\,. \tag{3.12}$$

The last equality follows from (3.9). Therefore $T^1 = 0$.

It remains to show $T^2 = 0$ as well. An inspection of the conditions we have derived above reveal that the only non-vanishing components are $(T^2_{\alpha\beta})_{\gamma\delta}$ and $(T^2_{\alpha\beta})_{\bar\gamma\bar\delta}$. The former vanishes because of the Bianchi identity of T^3, (2.12), involving skew-symmetry in two holomorphic and three anti-holomorphic indices, and the the relation of T^3 to T^2 in (3.6). To continue, first observe that from (3.9) and $T^1 = 0$, we find that

$$(T^2_{\alpha\gamma})_{\bar\beta}{}^{\gamma} = 0\ . \tag{3.13}$$

Next we shall use the expression of T^1 and T^3 in terms of the fluxes F which can be found in [11]. The condition $T^1 = 0$ implies that $F \wedge F = 0$ which in turn implies that

$$(T^3_{MN})_{PQR} = \tfrac{1}{6}(\nabla_M F_{NPQR} - \nabla_N F_{MPQR})\,. \tag{3.14}$$

Now consider the case where all five indices are holomorphic. This component of T^3 is subject to two additional conditions. The first follows from the Bianchi identity for the gauge field, which states that

$$(T^3_{[\alpha\beta_1})_{\beta_2\beta_3\beta_4]} = \tfrac{1}{15}(\nabla_\alpha F_{\beta_1\cdots\beta_4} + 4\nabla_{[\beta_1}F_{\beta_2\beta_3\beta_4]\alpha}) = 0\,. \tag{3.15}$$

The second condition follows from the relation between T^3 and T^2 in (3.6) and the trace condition on T^2 in (3.13). It implies that

$$(T^3_{\alpha[\beta_1})_{\beta_2\beta_3\beta_4]} = \tfrac{1}{6}(\nabla_\alpha F_{\beta_1\cdots\beta_4} + \nabla_{[\beta_1}F_{\beta_2\beta_3\beta_4]\alpha}) = 0\,. \tag{3.16}$$

Comparing (3.15) and (3.16), we deduce that $\nabla_\alpha F_{\beta_1\cdots\beta_4} = 0$. From this it follows that the T^3 component with five holomorphic indices vanishes, and this implies that $(T^2_{\alpha\beta})_{\bar\gamma\bar\delta} = 0$. Therefore $T^2 = 0$.

As we have already mentioned a direct inspection of (3.6) reveals that all T^k are determined in terms of T^1 and T^2. Thus $T^k = 0$ and so $\mathcal{R} = 0$. Therefore, the reduced holonomy of $N = 31$ backgrounds with an SU(5)-invariant normal is $\{1\}$, and so these backgrounds are locally isometric to the maximally supersymmetric ones.

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4. $(\mathrm{Spin}(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$-invariant normal

4.1 Integrability conditions

The null case can be investigated in a similar way. For this we use the null basis of [18] and choose the normal spinor as

$$\nu = 1 + e_{1234} \ . \tag{4.1}$$

A basis in the space of Majorana spinors orthogonal to ν is

$$\begin{aligned}
1 + e_{1234}, \quad i(1 - e_{1234})\,, \qquad & i(e_5 - e_{12345})\,, \\
e_\rho + \frac{1}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3}\,, \qquad & i(e_\rho - \frac{1}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3})\,, \\
e_{\rho 5} + \frac{1}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3 5}\,, \qquad & i(e_{\rho 5} - \frac{1}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3 5})\,, \\
i(e_{\rho_1\rho_2} + \frac{1}{2}\epsilon^{\rho_1\rho_2\mu_1\mu_2} e_{\mu_1\mu_2}), \qquad & e_{\rho_1\rho_2} - \frac{1}{2}\epsilon^{\rho_1\rho_2\mu_1\mu_2} e_{\mu_1\mu_2}\,, \\
i(e_{\rho_1\rho_2 5} + \frac{1}{2}\epsilon^{\rho_1\rho_2\mu_1\mu_2} e_{\mu_1\mu_2 5}), \qquad & e_{\rho_1\rho_2 5} - \frac{1}{2}\epsilon^{\rho_1\rho_2\mu_1\mu_2} e_{\mu_1\mu_2 5}\ .
\end{aligned} \tag{4.2}$$

For the analysis we shall present below, it is convenient to introduce a new SU(4)-covariant basis as

$$\begin{aligned}
\theta^\natural &= i(e_5 - e_{12345})\,, \qquad \theta^+ = i(1 - e_{1234})\,, \quad \theta^- = 1 + e_{1234}\,, \\
\theta^{-\rho} &= \frac{\sqrt{2}}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3}\,, \qquad \theta^{-\bar\rho} = \sqrt{2} e_\rho\,, \\
\theta^{\rho} &= \frac{\sqrt{2}}{3!}\epsilon^{\rho\sigma_1\sigma_2\sigma_3} e_{\sigma_1\sigma_2\sigma_3 5}\,, \qquad \theta^{\bar\rho} = \sqrt{2} e_{\rho 5}\,, \\
\theta^{-\bar\rho\bar\sigma} &= \sqrt{2} e_{\rho\sigma}\,, \qquad \theta^{\bar\rho\bar\sigma} = \sqrt{2} e_{\rho\sigma 5}\,, \quad \lambda,\mu,\nu,\rho,\sigma = 1,2,3,4\ .
\end{aligned} \tag{4.3}$$

It is then straightforward to show using (2.10) and the form bi-linears of appendix A that

$$\begin{aligned}
&u_\natural = 4i(T^2)_\mu{}^\mu\,, \quad u_- = -8\sqrt{2}(T^1)_-\,, \quad u_+ = 2\sqrt{2}i(T^3)_{-\mu}{}^\mu\,, \quad u_{-\rho} = 8\sqrt{2}(T^2)_{-\rho}\,, \\
&u_\rho = -16(T^1)_\rho\,, \quad \epsilon_{\rho\sigma}{}^{\bar\mu_1\bar\mu_2} u_{\bar\mu_1\bar\mu_2} = 8\sqrt{2}(T^2)_{\rho\sigma}\,, \quad \epsilon_{\rho\sigma}{}^{\bar\mu_1\bar\mu_2} u_{-\bar\mu_1\bar\mu_2} = 8(T^3)_{-\rho\sigma}\,,
\end{aligned} \tag{4.4}$$

where the two-form indices of u and T^k have been suppressed. In addition, we find that (2.10) implies the following relations between the T^k:

$$\begin{aligned}
(T^1)_+ = (T^1)_\natural &= 0\,, \\
(T^2)_{+-} = (T^2)_{+\rho} = (T^2)_{+\natural} &= 0\,, \\
(T^2)_{-\natural} &= (T^1)_-\,, \\
(T^2)_{\natural\rho} &= -(T^1)_\rho\,, \\
(T^2)_{\rho\bar\sigma} &= \frac{1}{4}(T^2)_\mu{}^\mu \delta_{\rho\bar\sigma}\,, \\
(T^2)_{\rho\sigma} + \frac{1}{2}\epsilon_{\rho\sigma}{}^{\bar\mu_1\bar\mu_2}(T^2)_{\bar\mu_1\bar\mu_2} &= 0\,, \\
(T^3)_{+-\rho} &= (T^1)_\rho\,, \\
(T^3)_{+-\natural} = (T^3)_{+\natural\rho} = (T^3)_{+\rho\sigma} = (T^3)_{+\rho\bar\sigma} &= 0\,,
\end{aligned}$$

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$$
\begin{aligned}
(T^3)_{-\natural\rho} &= -(T^2)_{-\rho}\,,\\
(T^3)_{-\rho\bar\sigma} &= \frac{1}{4}(T^3)_{-\mu}{}^{\mu}\delta_{\rho\bar\sigma}\,,\\
(T^3)_{\natural\rho\sigma} &= (T^2)_{\rho\sigma}\,,\\
(T^3)_{\natural\rho\bar\sigma} &= \frac{1}{4}(T^2)_{\mu}{}^{\mu}\delta_{\rho\bar\sigma}\,,\\
(T^3)_{\sigma_1\sigma_2\sigma_3} &= -2\epsilon_{\sigma_1\sigma_2\sigma_3}{}^{\bar\rho}(T^1)_{\bar\rho}\,,\\
(T^3)_{\sigma_1\sigma_2\bar\rho} &= 2\delta_{\bar\rho[\sigma_1}(T^1)_{\sigma_2]}\,,\\
(T^3)_{-\rho\sigma}+\frac{1}{2}\epsilon_{\rho\sigma}{}^{\bar\mu_1\bar\mu_2}(T^3)_{-\bar\mu_1\bar\mu_2} &= 0\,,\\
(T^4)_{+-\natural\rho} &= -(T^1)_{\rho}\,,\\
(T^4)_{+-\rho\sigma} &= (T^2)_{\rho\sigma}\,,\\
(T^4)_{+-\rho\bar\sigma} &= \frac{1}{4}(T^2)_{\mu}{}^{\mu}\delta_{\rho\bar\sigma}\,,\\
(T^4)_{+\natural\rho\sigma}=(T^4)_{+\natural\rho\bar\sigma}=(T^4)_{+\sigma_1\sigma_2\sigma_3}=(T^4)_{+\sigma_1\sigma_2\bar\rho} &= 0\,,\\
(T^4)_{-\natural\rho\sigma} &= (T^3)_{-\rho\sigma}\,,\\
(T^4)_{-\natural\rho\bar\sigma} &= \frac{1}{4}(T^3)_{-\mu}{}^{\mu}\delta_{\rho\bar\sigma}\,,\\
(T^4)_{-\sigma_1\sigma_2\sigma_3} &= 2(T^2)_{-\bar\rho}\epsilon^{\bar\rho}{}_{\sigma_1\sigma_2\sigma_3}\,,\\
(T^4)_{-\sigma_1\sigma_2\bar\rho} &= 2\delta_{\bar\rho[\sigma_1}(T^2)_{|-|\sigma_2]}\,,\\
(T^4)_{\natural\sigma_1\sigma_2\sigma_3} &= -2(T^1)_{\bar\rho}\epsilon^{\bar\rho}{}_{\sigma_1\sigma_2\sigma_3}\,,\\
(T^4)_{\natural\sigma_1\sigma_2\bar\rho} &= -2\delta_{\bar\rho[\sigma_1}(T^1)_{\sigma_2]}\,,\\
(T^4)_{\sigma_1\sigma_2\sigma_3\sigma_4} &= \frac{1}{2}(T^2)_{\mu}{}^{\mu}\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,,\\
(T^4)_{\sigma_1\sigma_2\sigma_3\bar\rho} &= -3\delta_{\bar\rho[\sigma_1}(T^2)_{\sigma_2\sigma_3]}\,,\\
(T^4)_{\rho_1\rho_2\bar\sigma_1\bar\sigma_2} &= 0\,,\\
(T^5)_{+-\natural\rho\sigma} &= (T^2)_{\rho\sigma}\,,\\
(T^5)_{+-\natural\rho\bar\sigma} &= \frac{1}{4}(T^2)_{\mu}{}^{\mu}\delta_{\rho\bar\sigma}\,,\\
(T^5)_{+-\sigma_1\sigma_2\sigma_3} &= 2(T^1)_{\bar\rho}\epsilon^{\bar\rho}{}_{\sigma_1\sigma_2\sigma_3}\,,\\
(T^5)_{+-\sigma_1\sigma_2\bar\rho} &= 2\delta_{\bar\rho[\sigma_1}(T^1)_{\sigma_2]}\,,\\
(T^5)_{+\natural\sigma_1\sigma_2\sigma_3}=(T^5)_{+\natural\sigma_1\sigma_2\bar\rho} &= 0\,,\\
(T^5)_{+\sigma_1\sigma_2\sigma_3\sigma_4}=(T^5)_{+\sigma_1\sigma_2\sigma_3\bar\rho}=(T^5)_{+\sigma_1\sigma_2\bar\rho_1\bar\rho_2} &= 0\,,\\
(T^5)_{-\natural\sigma_1\sigma_2\sigma_3} &= -2(T^2)_{-\bar\rho}\epsilon^{\bar\rho}{}_{\sigma_1\sigma_2\sigma_3}\,,\\
(T^5)_{-\natural\sigma_1\sigma_2\bar\rho} &= -2\delta_{\bar\rho[\sigma_1}(T^2)_{|-|\sigma_2]}\,,\\
(T^5)_{-\sigma_1\sigma_2\sigma_3\sigma_4} &= (\frac{1}{2}(T^3)_{-\mu}{}^{\mu}+2(T^1)_{-})\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,,\\
(T^5)_{-\sigma_1\sigma_2\sigma_3\bar\rho} &= -3\delta_{\bar\rho[\sigma_1}(T^3)_{|-|\sigma_2\sigma_3]}\,,\\
(T^5)_{-\sigma_1\sigma_2\bar\rho_1\bar\rho_2} &= -2\delta_{\sigma_1[\bar\rho_1}\delta_{\bar\rho_2]\sigma_2}(T^1)_{-}\,,\\
(T^5)_{\natural\sigma_1\sigma_2\sigma_3\sigma_4} &= \frac{1}{2}(T^2)_{\mu}{}^{\mu}\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,,\\
(T^5)_{\natural\sigma_1\sigma_2\sigma_3\bar\rho} &= -3\delta_{\bar\rho[\sigma_1}(T^2)_{\sigma_2\sigma_3]}\,,\\
(T^5)_{\natural\sigma_1\sigma_2\bar\rho_1\bar\rho_2} &= 0\,,\\
(T^5)_{\sigma_1\sigma_2\sigma_3\sigma_4\bar\rho} &= 2\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}(T^1)_{\bar\rho}\,,\\
(T^5)_{\sigma_1\sigma_2\sigma_3\bar\rho_1\bar\rho_2} &= -6\delta_{\bar\rho_1[\sigma_1}\delta_{\sigma_2|\bar\rho_2}(T^1)_{\sigma_3]}\,. \qquad (4.5)
\end{aligned}
$$

Observe that all components T^k of the supercurvature $\mathcal{R}$ are determined in terms of T^1, T^2 and T^3.

4.2 Solving the conditions

We shall now use (2.11), (2.12) and the explicit expressions of T^k in terms of the physical fields which can be found in [11] to show that $\mathcal{R} = 0$. Since all the components of $\mathcal{R}$ in this case depend of T^1, T^2 and T^3, let us first show that $T^1 = 0$. Due to (4.5) and the skew-symmetry of $(T^1_{MN})_P$, the only possible non-vanishing components of T^1 up to complex conjugation are $(T^1_{\rho_1\rho_2})_{\rho_3}$, $(T^1_{\rho_1\rho_2})_{\bar\sigma}$, $(T^1_{\rho\sigma})_-$, $(T^1_{\rho\bar\sigma})_-$.

First consider the condition on T^2 in (2.12). Taking $Q = \natural$, this implies that T^1 satisfies

$$(T^1_{\rho_1\rho_2})_\sigma = (T^1_{\rho_1\rho_2})_{\bar\sigma} = 0, \quad (T^1_{\rho_1\rho_2})_- = -\tfrac{1}{2}\epsilon_{\rho_1\rho_2}{}^{\bar\sigma_1\bar\sigma_2}(T^1_{\bar\sigma_1\bar\sigma_2})_-, \quad (T^1_{\rho\bar\sigma})_- = \tfrac{1}{4}(T^{1\lambda}_\lambda)_- g_{\rho\bar\sigma}. \tag{4.6}$$

Next turn to the conditions in (2.11). From $(T^2_{\natural[-})_{\rho\bar\sigma]} - \frac{1}{3}(T^4_{\natural N})_{-\rho\bar\sigma}{}^N = 0$, we find

$$(T^1_{\rho\bar\sigma})_- = -(T^{1\mu}_\mu)_-\delta_{\rho\bar\sigma}\,, \tag{4.7}$$

which implies that

$$(T^1_{\rho\bar\sigma})_- = 0\,. \tag{4.8}$$

In addition $(T^1_{\rho_1\rho_2})_- + \frac{1}{2}(T^3_{\rho_1 N})_{\rho_2 -}{}^N = 0$ implies that

$$(T^3_{\rho_1\bar\lambda})^{\bar\lambda}{}_{\rho_2 -} = -2(T^1_{\rho_1\rho_2})_- + \frac{1}{4}(T^3_{\rho_1\rho_2})_{-\mu}{}^\mu\,. \tag{4.9}$$

Combining this result with the condition $(T^2_{\rho[\sigma})_{-\natural]} - \frac{1}{3}(T^4_{\rho N})_{\sigma-\natural}{}^N = 0$, which yields

$$(T^3_{\rho_1\bar\lambda})^{\bar\lambda}{}_{\rho_2 -} = -4(T^1_{\rho_1\rho_2})_- + \frac{1}{4}(T^3_{\rho_1\rho_2})_{-\mu}{}^\mu\,, \tag{4.10}$$

we find $(T^1_{\rho\sigma})_- = 0$. Hence $T^1 = 0$.

We now turn our attention to T^2. From (4.5) and the symmetry property in (2.12), it follows that $(T^2_{MN})_{P+} = (T^2_{P+})_{MN} = (T^2_{P\natural})_{MN} = 0$. Furthermore, $(T^2_{MN})_{\rho\sigma} = (T^2_{\rho\sigma})_{MN}$ are self-dual, and $(T^2_{MN})_{\rho\bar\sigma} = (T^2_{\rho\bar\sigma})_{MN}$ are determined in terms of the trace.

Let us first consider the case where all four indices are of SU(4) type. From $(T^2_{\rho N})_{\bar\sigma}{}^N = 0$ and $(T^3_{[\rho_1\rho_2})_{\bar\sigma_1\bar\sigma_2\natural]} = 0$, we find respectively

$$(T^2_{\rho\lambda})_{\bar\sigma}{}^\lambda = \tfrac{1}{16}g_{\rho\bar\sigma}(T^2{}_\sigma{}^\sigma)_\lambda{}^\lambda\,, \quad (T^2_{\rho_1\rho_2})_{\bar\sigma_1\bar\sigma_2} = -2(T^2_{\bar\sigma_1[\rho_1})_{a_2]\bar\sigma_2}\,. \tag{4.11}$$

By taking the trace of the second equation, we conclude that these expressions vanish. Hence the equations imply that $(T^2_{\rho_1\rho_2})_{\bar\sigma_1\bar\sigma_2} = (T^2_{\rho\bar\sigma})_{\lambda\bar\delta} = 0$. Furthermore, $(T^3_{[\rho_1\rho_2})_{\rho_3\bar\sigma_1\natural]} = 0$ implies that $(T^2_{\rho\sigma})_{\lambda\bar\delta} = 0$. Therefore T^2 with only SU(4) indices vanishes.

Next we consider the case where one of the indices equals $-$. From $(T^2_{\rho N})_-{}^N = 0$ and $(T^2_{-[\rho_1})_{\rho_2\rho_3]} - \frac{1}{3}(T^4_{-N})_{\rho_1\rho_2\rho_3}{}^N = 0$, we find that

$$(T^2_{-\rho})_\sigma{}^\sigma = -4(T^2_{\rho\lambda})_-{}^\lambda\,, \quad (T^2_{-[\rho_1})_{\rho_2\rho_3]} = \tfrac{1}{12}\epsilon_{\rho_1\rho_2\rho_3}{}^{\bar\sigma}(T^2_{-\bar\sigma})_\lambda{}^\lambda\,. \tag{4.12}$$

In addition, we explore the relations of T^3 which arise from $(T^3_{[\natural\rho_1})_{\rho_2\rho_3-]} = 0$, $(T^3_{[\natural\bar\sigma})_{\rho_1\rho_2-]} = 0$, $(T^2_{\rho[-})_{\sigma_1\sigma_2]} - \frac{1}{3}(T^4_{\rho N})_{-\sigma_1\sigma_2}{}^N = 0$ and $(T^2_{\rho[-})_{\bar\sigma_1\bar\sigma_2]} - \frac{1}{3}(T^4_{\rho N})_{-\bar\sigma_1\bar\sigma_2}{}^N = 0$ to find

$$(T^3_{\natural[\rho_1})_{\rho_2\rho_3]-} = 2(T^2_{-[\rho_1})_{\rho_2\rho_3]}\,,$$

$$
\begin{aligned}
(T^3_{\natural\bar\sigma})_{\rho_1\rho_2-} &= 4(T^2_{-[\rho_1})_{\rho_2]\bar\sigma} + 2(T^2_{-\bar\sigma})_{\rho_1\rho_2} - 2(T^3_{\natural[\rho_1})_{\rho_2]\bar\sigma-}\,,\\
(T^3_{\rho\natural})_{-\sigma_1\sigma_2} &= 2(T^2_{\rho-})_{\sigma_1\sigma_2} - 4(T^2_{\rho[\sigma_1})_{|-|\sigma_2]} - \tfrac{1}{2}(T^2_{-\bar\lambda})_\delta{}^\delta\epsilon^{\bar\lambda}{}_{\rho\sigma_1\sigma_2}\,,\\
(T^3_{\rho\natural})_{-\bar\sigma_1\bar\sigma_2} &= 2(T^2_{\rho-})_{\bar\sigma_1\bar\sigma_2} - 4(T^2_{\rho[\bar\sigma_1})_{|-|\bar\sigma_2]} - 2(T^2_{\rho\lambda})_{-\delta}\epsilon_{\bar\sigma_1\bar\sigma_2}{}^{\lambda\delta}\,.
\end{aligned}
\tag{4.13}
$$

From the two expressions above with three holomorphic indices it follows that

$$
(T^2_{-[\rho_1})_{\rho_2\rho_3]} = \tfrac{1}{8}\epsilon_{\rho_1\rho_2\rho_3}{}^{\bar\sigma}(T^2_{-\bar\sigma})_\lambda{}^\lambda\,. \tag{4.14}
$$

Combining this with (4.12), we conclude that these expressions vanish, and therefore $(T^2_{-\rho})_{\sigma\bar\lambda} = 0$. Then, the definition for $(T^3_{\natural\bar\sigma})_{\rho_1\rho_2-}$ and its complex conjugate imply that $(T^2_{-\rho})_{\sigma\lambda} = 0$. Therefore T^2 with three SU(4) indices also vanishes.

The only remaining non-vanishing components are $(T^2_{-\rho})_{-\sigma}$ and $(T^2_{-\rho})_{-\bar\sigma}$. First, note that $(T^2_{-\rho})_{-\sigma}$ is symmetric in the interchange of ρ and σ, while in terms of F it is given by

$$
(T^2_{-\rho})_{-\sigma} = (T^3_{-\rho})_{-\sigma\natural} = \tfrac{1}{6}\nabla_- F_{\rho-\sigma\natural}\,, \tag{4.15}
$$

which is skew-symmetric in the interchange and so $(T^2_{-\rho})_{-\sigma} = 0$. Similarly, $(T^2_{-\rho})_{-\bar\sigma} = (T^2_{-\bar\sigma})_{-\rho}$ while

$$
(T^2_{-\rho})_{-\bar\sigma} = (T^3_{-\rho})_{-\bar\sigma\natural} = \tfrac{1}{6}\nabla_- F_{\rho-\bar\sigma\natural} = -\tfrac{1}{6}\nabla_- F_{\bar\sigma-\rho\natural} = -(T^2_{-\bar\sigma})_{-\rho}\,. \tag{4.16}
$$

Hence this component also vanishes. Therefore we conclude that $T^2 = 0$.

It remains to consider T^3, and in particular the components $(T^3)_{\mu\nu-}$ and $(T^3)_{\mu\bar\nu-}$. The vanishing of $(T^3_{MN})_{\mu\nu}{}^N$ for $M = -, \natural, \rho, \bar\rho$ implies that

$$
(T^3_{+-})_{\mu\nu-} = (T^3_{+\natural})_{\mu\nu-} = (T^3_{+\rho})_{\mu\nu-} = (T^3_{+\bar\rho})_{\mu\nu-} = 0\,. \tag{4.17}
$$

From the vanishing of $(T^4_{MN})_{-\mu\nu}{}^N$ for $M = -, \rho, \bar\rho$, we also get

$$
(T^3_{-\natural})_{\mu\nu-} = (T^3_{\natural\rho})_{\mu\nu-} = (T^3_{\natural\bar\rho})_{\mu\nu-} = 0\,. \tag{4.18}
$$

Next, note that

$$
(T^3_{-\rho})_{-\bar\sigma_1\bar\sigma_2} = (T^3_{-\bar\sigma_1})_{-\bar\sigma_2\rho} = \frac{1}{4}(T^3_{-\bar\sigma_1})_\rho{}^\rho{}_- g_{\rho\bar\sigma_2}\,, \tag{4.19}
$$

and on symmetrizing this expression in σ_1, σ_2 and taking the trace, we find $(T^3_{-\bar\sigma_1})_\mu{}^\mu{}_- = 0$ and hence $(T^3_{-\rho})_{-\bar\sigma_1\bar\sigma_2} = 0$.

Combining the Bianchi identity for F with

$$
(T^3_{-\rho_1})_{\rho_2\bar\sigma_1\bar\sigma_2} = \tfrac{1}{6}(\nabla_- F_{\rho_1\rho_2\bar\sigma_1\bar\sigma_2} - \nabla_{\rho_1}F_{-\rho_2\bar\sigma_1\bar\sigma_2}) = 0\,, \tag{4.20}
$$

we find that $\nabla_{\rho_1}F_{-\rho_2\bar\sigma_1\bar\sigma_2} = 0$ and hence $(T^3_{\rho_1\rho_2})_{-\bar\sigma_1\bar\sigma_2}$ vanishes. $(T^3_{\rho_1\rho_2})_{\sigma_1\sigma_2-} = 0$ due to the duality condition in (4.5). Finally, the Bianchi identity for F together with

$$
\begin{aligned}
(T^3_{-\rho})_{\lambda_1\lambda_2\bar\sigma} &= \tfrac{1}{6}(\nabla_- F_{\rho\lambda_1\lambda_2\bar\sigma} - \nabla_\rho F_{-\lambda_1\lambda_2\bar\sigma}) = 0\,,\\
(T^3_{\bar\sigma-})_{\rho\lambda_1\lambda_2} &= \tfrac{1}{6}(\nabla_{\bar\sigma} F_{-\rho\lambda_1\lambda_2} - \nabla_- F_{\bar\sigma\rho\lambda_1\lambda_2}) = 0\,,
\end{aligned}
\tag{4.21}
$$

imply that $\nabla_\rho F_{-\mu\nu\bar{\sigma}} = \nabla_{\bar{\sigma}} F_{-\rho\mu\nu} = 0$, and hence $(T^3_{\rho\bar{\sigma}})_{-\lambda_1\lambda_2} = 0$. Hence $(T^3)_{\mu\nu-} = 0$.

In order to show that the remaining components of $(T^3)_{\mu\bar{\nu}-}$ also vanish, note that $(T^3_{MN})_{\mu\bar{\nu}}{}^N = 0$ for $M = -, \natural, \rho, \bar{\rho}$ in (2.11) implies

$$(T^3_{+-})_{\mu\bar{\nu}-} = (T^3_{+\natural})_{\mu\bar{\nu}-} = (T^3_{+\rho})_{\mu\bar{\nu}-} = (T^3_{+\bar{\rho}})_{\mu\bar{\nu}-} = 0\,, \tag{4.22}$$

and the vanishing of $(T^4_{MN})_{-\mu\bar{\nu}}{}^N$ for $M = -, \rho$ implies

$$(T^3_{-\natural})_{\mu\bar{\nu}-} = (T^3_{\natural\rho})_{\mu\bar{\nu}-} = 0\ . \tag{4.23}$$

Next, as we have shown that $\nabla_\rho F_{-\mu\nu\bar{\sigma}} = 0$, this implies $(T^3_{\rho_1\rho_2})_{\mu\bar{\nu}-} = 0$, and hence $(T^3_{\bar{\rho}_1\bar{\rho}_2})_{\mu\bar{\nu}-} = 0$. Also, $(T^3_{-\bar{\sigma}_1})_{-\bar{\sigma}_2\rho} = 0$ from (4.19). Lastly, by taking traces of the constraint $(T^3_{[\rho_1\bar{\rho}_2})_{\sigma_1\bar{\sigma}_2-]} = 0$ and using $(T^3_{\rho_1\sigma_1})_{\bar{\rho}_2\bar{\sigma}_2-} = (T^3_{\bar{\rho}_2\bar{\sigma}_2})_{\rho_1\sigma_1-} = 0$, we find $(T^3_{\rho_1\bar{\rho}_2})_{\sigma_1\bar{\sigma}_2-} = 0$. Hence $(T^3)_{\mu\bar{\nu}-} = 0$. These conditions are then sufficient to show that $T^3 = 0$.

As we have already mentioned, T^k are determined from T^1, T^2 and T^3. Since $T^1 = T^2 = T^3 = 0$, $T^k = 0$ and so $\mathcal{R} = 0$. Therefore, the reduced holonomy of $N = 31$ backgrounds with a $(\mathrm{Spin}(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$-invariant normal is $\{1\}$, and so these backgrounds are locally isometric to maximally supersymmetric ones. Combining this result with that of the previous section section, we conclude that all $N = 31$ backgrounds of eleven-dimensional supergravity admit locally an additional Killing spinor and so they are maximally supersymmetric.

5. $N = 15$ in type I supergravities

The non-existence of $N = 15$ supersymmetric backgrounds in type I supergravities can be easily seen by combining the results of [3] and [17]. In particular, the normal to the 15 Killing spinors has stability subgroup $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$. So there is only one case to consider. It is convenient to choose

$$\nu = e_2 - e_{134}\ . \tag{5.1}$$

Then combining the conditions of the backgrounds with Killing spinors that have stability subgroup $\mathbb{R}^8$ and those that have stability subgroup G_2 in [17], one finds that the dilaton Φ is constant and the non-vanishing components of H are H_{-ij}, where $i, j = 1, \ldots 8$. The dilatino Killing spinor equation becomes

$$H_{-ij}\Gamma^{-ij}\epsilon^r = 0\ . \tag{5.2}$$

The existence of a non-trivial solution for this equation is equivalent to requiring that there are seven linearly independent spinors in the chiral or anti-chiral representation of Spin(8), depending on conventions, with a non-trivial stability subgroup. This is not the case and so $H = 0$. Similarly, the integrability condition of the gravitino Killing spinor equation implies that the supercovariant curvature of the connection with torsion vanishes, $\hat{R} = 0$. Since $H = 0$, $\hat{R} = R = 0$, the Riemann curvature of the spacetime vanishes. The rest of the fluxes, e.g. gauge field strengths, can also be shown to vanish. Therefore, the spacetime is locally isometric to Minkowski space with constant dilaton, and vanishing three-form and gauge field fluxes.

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6. Concluding remarks

We have shown that eleven-dimensional supergravity backgrounds with 31 supersymmetries are locally isometric to maximally supersymmetric ones. This result together with that of [11] (locally) classify the supersymmetric backgrounds of eleven-dimensional supergravity with $N = 31$ and $N = 32$ supersymmetries. The Killing spinor equations of eleven-dimensional supergravity for the $N = 1$ backgrounds have been solved in [23]. So far, these are the only three cases in eleven-dimensions that the geometry of the backgrounds has been identified for a given N. Furthermore, the result of this paper together with those obtained in [3] and [4] rule out the existence of $N = 31$ solutions in eleven- and type II ten-dimensional supergravities. In addition, a straightforward argument can rule out the existence of $N = 15$ backgrounds in type I ten-dimensional supergravities. In lower-dimensions, a similar conclusion has been reached for the cases that have been investigated in [5]. There are many more lower dimensional cases that can be explored.

It is clear from the cases that have been examined so far that backgrounds with $N_{\text{max}}-1$ number of supersymmetries are severely restricted. This raises the possibility that there are much less supersymmetric backgrounds in ten and eleven dimensions than those that may have been expected from the holonomy argument of [8 − 10, 19]. In the proof that the $N = 31$ eleven-dimensional backgrounds admit 32 supersymmetries, we have used both the conditions that arise from the Killing spinor equations as well as field equations and Bianchi identities. It has been the field equations and Bianchi identities that enforced the condition that the supercovariant curvature vanishes — the conditions arising from the Killing spinor equations were not sufficient. Dynamical information has been necessary to construct the proof. This is unlike the type II theories where the Killing spinor equations were sufficient to establish the result.

Another property of the $N = 31$ backgrounds in eleven or ten dimensions is that the stability subgroup of Killing spinors in $\mathrm{Spin}(10, 1)$ or $\mathrm{Spin}(9, 1)$ is trivial, i.e. $\mathrm{stab}(\epsilon) = \{1\}$. These are the first examples, other than those with maximal supersymmetry, that have this property. It is encouraging that it turned out to be that such backgrounds are in fact maximally supersymmetric. This may suggest that even backgrounds with a small number of Killing spinors but with a trivial stability subgroup in the gauge group of the Killing spinor equations are severely restricted, though it is possible that such new backgrounds exist. If this is the case, the classification of supersymmetric backgrounds in ten and eleven dimensions may be somewhat simplified. It would be worth investigating more such examples in the future.

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A. Spacetime form spinor bi-linears

In the computation of the conditions that arise from the integrability condition $\mathcal{R}\epsilon^r = 0$, we have used the form spinor bi-linears of the SU(5)-invariant normal spinor ν and a basis θ^r, (3.4), that spans the 31 Killing spinors. These bi-linears are defined as

$$\tau^r = \frac{1}{k!} B(\theta^r, \Gamma_{A_1 A_2 \dots A_k} \nu)\, e^{A_1} \wedge e^{A_2} \wedge \cdots \wedge e^{A_k} \ . \tag{A.1}$$

In particular the non-vanishing components of the one-forms are

$$\tau_\alpha^\beta = 2\delta_\alpha^\beta \,, \quad \tau_0^0 = -2 \ . \tag{A.2}$$

The two-forms are

$$\tau_{0\alpha}^\beta = -2i\delta_\alpha^\beta \,, \quad \tau_{\alpha\bar\beta}^0 = -2ig_{\alpha\bar\beta} \,, \quad \tau_{\alpha\beta}^{\gamma\delta} = 4\sqrt{2}i\delta_{[\alpha\beta]}^{\gamma\delta} \ . \tag{A.3}$$

The three-forms are

$$\tau_{0\alpha\beta}^{\gamma\delta} = -4\sqrt{2}\delta_{[\alpha\beta]}^{\gamma\delta} \,, \quad \tau_{\gamma_1\gamma_2\gamma_3}^{\bar\alpha\bar\beta} = 4i\epsilon_{\gamma_1\gamma_2\gamma_3}{}^{\bar\alpha\bar\beta} \,, \quad \tau_{\alpha\beta\bar\gamma}^{\delta} = -4\delta_{[\alpha}^{\delta} g_{\beta]\bar\gamma} \ . \tag{A.4}$$

The four-forms are

$$\begin{aligned} \tau_{0\gamma_1\gamma_2\gamma_3}^{\bar\alpha\bar\beta} &= 4\epsilon_{\gamma_1\gamma_2\gamma_3}{}^{\bar\alpha\bar\beta} \,, \quad \tau_{0\beta_1\beta_2\bar\gamma}^{\alpha} = 4i\delta_{[\beta_1}^{\alpha} g_{\beta_2]\bar\gamma} \,, \quad \tau_{\beta_1\beta_2\beta_3\beta_4}^{\bar\alpha} = 4\sqrt{2}\epsilon_{\beta_1\beta_2\beta_3\beta_4}{}^{\bar\alpha} \,, \\ \tau_{\gamma_1\gamma_2\gamma_3\bar\gamma_4}^{\alpha\beta} &= -12\sqrt{2}i\delta_{[\gamma_1\gamma_2}^{\alpha\beta} g_{\gamma_3]\bar\gamma_4} \,, \end{aligned} \tag{A.5}$$

and the five-forms are

$$\begin{aligned} \tau_{0\beta_1\beta_2\beta_3\beta_4}^{\bar\alpha} &= 4\sqrt{2}i\epsilon_{b_1\beta_2\beta_3\beta_4}{}^{\bar\alpha} \,, \qquad \tau_{0\gamma_1\gamma_2\gamma_3\bar\gamma_4}^{\alpha\beta} = 12\sqrt{2}\delta_{[\gamma_1\gamma_2}^{\alpha\beta} g_{\gamma_3]\bar\gamma_4} \,, \quad \tau_{0\alpha\beta\bar\gamma\bar\delta}^{0} = 4g_{\alpha[\bar\gamma}g_{|\beta|\bar\delta]} \,, \\ \tau_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}^{0} &= -4\sqrt{2}i\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} \,, \quad \tau_{\beta_1\beta_2\beta_3\beta_4\bar\gamma}^{\bar\alpha_1\bar\alpha_2} = 8i\epsilon_{\beta_1\beta_2\beta_3\beta_4}{}^{[\bar\alpha_1}\delta_{\bar\gamma}^{\bar\alpha_2]} \,, \\ \tau_{\beta_1\beta_2\beta_3\bar\gamma_1\bar\gamma_2}^{\alpha} &= -12\delta_{[\beta_1}^{\alpha} g_{\beta_2|\bar\gamma_1|} g_{\beta_3]\bar\gamma_2} \,, \end{aligned} \tag{A.6}$$

where we have used $\delta_{[\alpha_1\alpha_2]}^{\beta_1\beta_2} = \delta_{[\alpha_1}^{\beta_1}\delta_{\alpha_2]}^{\beta_2}$.

Similarly, in the computation of the integrability conditions of $N = 31$ backgrounds with a $(\mathrm{Spin}(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$-invariant normal spinor ν, we have used the spacetime form spinor bi-linears of ν with the elements of the spinor basis (4.3). In particular, we find that the one-forms are

$$\tau^-{}_- = -2\sqrt{2}, \quad \tau^\rho{}_\sigma = -2\delta_\sigma^\rho \,, \tag{A.7}$$

the two-forms are

$$\begin{aligned} &\tau^\natural{}_{\rho\bar\sigma} = 2i\delta_{\rho\bar\sigma}, \qquad \tau^-{}_{-\natural} = 2\sqrt{2}, \qquad \tau^{-\rho}{}_{-\sigma} = -2\sqrt{2}\delta_\sigma^\rho \,, \\ &\tau^\rho{}_{\natural\sigma} = -2\delta_\sigma^\rho, \quad \tau^{\bar\rho\bar\sigma}{}_{\bar\mu_1\bar\mu_2} = 4\sqrt{2}\delta_{\bar\mu_1\bar\mu_2}^{\bar\rho\bar\sigma}, \quad \tau^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2} = -2\sqrt{2}\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2} \,, \end{aligned} \tag{A.8}$$

the three-forms are

$$\begin{aligned} &\tau^\natural{}_{\natural\rho\bar\sigma} = -2i\delta_{\rho\bar\sigma}, \qquad \tau^+{}_{-\rho\bar\sigma} = -2\sqrt{2}i\delta_{\rho\bar\sigma}, \qquad \tau^{-\rho}{}_{-\natural\sigma} = -2\sqrt{2}\delta_\sigma^\rho \,, \\ &\tau^\rho{}_{+-\sigma} = -2\delta_\sigma^\rho, \qquad \tau^\rho{}_{\sigma_1\sigma_2\bar\lambda} = -4\delta_{\bar\lambda[\sigma_1}\delta_{\sigma_2]}^\rho, \quad \tau^\rho{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3} = -4\epsilon^\rho{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3} \,, \end{aligned}$$

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$$
\begin{aligned}
\tau^{-\bar\rho\bar\sigma}{}_{-\bar\mu_1\bar\mu_2} &= -8\delta^{\bar\rho\bar\sigma}_{\bar\mu_1\bar\mu_2}, & \tau^{-\bar\rho\bar\sigma}{}_{-\mu_1\mu_2} &= 4\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2}\,, \\
\tau^{\bar\rho\bar\sigma}{}_{\natural\bar\mu_1\bar\mu_2} &= -4\sqrt{2}\delta^{\bar\rho\bar\sigma}_{\bar\mu_1\bar\mu_2}, & \tau^{\bar\rho\bar\sigma}{}_{\natural\mu_1\mu_2} &= 2\sqrt{2}\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2}\,,
\end{aligned}
\tag{A.9}
$$

the four-forms are

$$
\begin{aligned}
\tau^{\natural}{}_{+-\rho\bar\sigma} &= 2i\delta_{\rho\bar\sigma}, & \tau^{\natural}{}_{\rho_1\rho_2\rho_3\rho_4} &= 4i\epsilon_{\rho_1\rho_2\rho_3\rho_4}, & \tau^{+}{}_{-\natural\rho\bar\sigma} &= 2\sqrt{2}i\delta_{\rho\bar\sigma}\,, \\
\tau^{-\rho}{}_{-\sigma_1\sigma_2\bar\lambda} &= -4\sqrt{2}\delta_{\bar\lambda[\sigma_1}\delta^{\rho}_{\sigma_2]}, & \tau^{-\rho}{}_{-\bar\sigma_1\bar\sigma_2\bar\sigma_3} &= -4\sqrt{2}\epsilon^{\rho}{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3}\,, \\
\tau^{\rho}{}_{+-\natural\sigma} &= -2\delta^{\rho}_{\sigma}, & \tau^{\rho}{}_{\natural\sigma_1\sigma_2\bar\lambda} &= -4\delta_{\bar\lambda[\sigma_1}\delta^{\rho}_{\sigma_2]}, & \tau^{\rho}{}_{\natural\bar\sigma_1\bar\sigma_2\bar\sigma_3} &= -4\epsilon^{\rho}{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3}\,, \\
\tau^{-\bar\rho\bar\sigma}{}_{-\natural\bar\mu_1\bar\mu_2} &= 8\delta^{\bar\rho\bar\sigma}_{\bar\mu_1\bar\mu_2}, & \tau^{-\bar\rho\bar\sigma}{}_{-\natural\mu_1\mu_2} &= -4\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2}\,, \\
\tau^{\bar\rho\bar\sigma}{}_{+-\bar\mu_1\bar\mu_2} &= 4\sqrt{2}\delta^{\bar\rho\bar\sigma}_{\bar\mu_1\bar\mu_2}, & \tau^{\bar\rho\bar\sigma}{}_{+-\mu_1\mu_2} &= -2\sqrt{2}\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2}\,, \\
\tau^{\bar\rho\bar\sigma}{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3\lambda} &= -12\sqrt{2}\delta_{\lambda[\bar\sigma_1}\delta^{\bar\rho}_{\bar\sigma_2}\delta^{\bar\sigma}_{\bar\sigma_3]}, & \tau^{\bar\rho\bar\sigma}{}_{\bar\lambda\sigma_1\sigma_2\sigma_3} &= 4\sqrt{2}\epsilon_{\sigma_1\sigma_2\sigma_3}{}^{[\bar\rho}\delta^{\bar\sigma]}_{\bar\lambda}\,,
\end{aligned}
\tag{A.10}
$$

and the five-forms are

$$
\begin{aligned}
{}^{\natural}{}_{+-\natural\rho\bar\sigma} &= -2i\delta_{\rho\bar\sigma}, & \tau^{\natural}{}_{\natural\sigma_1\sigma_2\sigma_3\sigma_4} &= -4i\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,, \\
\tau^{-}{}_{-\sigma_1\sigma_2\bar\rho_1\bar\rho_2} &= 4\sqrt{2}\delta_{\sigma_1[\bar\rho_1}\delta_{\bar\rho_2]\sigma_2}, & \tau^{+}{}_{-\sigma_1\sigma_2\sigma_3\sigma_4} &= -4\sqrt{2}i\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,, \\
\tau^{-}{}_{-\sigma_1\sigma_2\sigma_3\sigma_4} &= -4\sqrt{2}\epsilon_{\sigma_1\sigma_2\sigma_3\sigma_4}\,, \\
\tau^{-\rho}{}_{-\natural\sigma_1\sigma_2\bar\lambda} &= -4\sqrt{2}\delta_{\bar\lambda[\sigma_1}\delta^{\rho}_{\sigma_2]}, & \tau^{-\rho}{}_{-\natural\bar\sigma_1\bar\sigma_2\bar\sigma_3} &= -4\sqrt{2}\epsilon^{\rho}{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3}\,, \\
\tau^{\rho}{}_{+-\sigma_1\sigma_2\bar\lambda} &= -4\delta_{\bar\lambda[\sigma_1}\delta^{\rho}_{\sigma_2]}, & \tau^{\rho}{}_{+-\bar\sigma_1\bar\sigma_2\bar\sigma_3} &= -4\epsilon^{\rho}{}_{\bar\sigma_1\bar\sigma_2\bar\sigma_3}\,, \\
\tau^{\rho}{}_{\sigma_1\sigma_2\sigma_3\bar\rho_1\bar\rho_2} &= 12\delta_{\bar\rho_1[\sigma_1}\delta_{\sigma_2|\bar\rho_2|}\delta^{\rho}_{\sigma_3]}, & \tau^{\rho}{}_{\lambda\bar\sigma_1\bar\sigma_2\bar\sigma_3\bar\sigma_4} &= -4\delta^{\rho}_{\lambda}\epsilon_{\bar\sigma_1\bar\sigma_2\bar\sigma_3\bar\sigma_4}\,, \\
\tau^{-\bar\rho\bar\sigma}{}_{-\bar\sigma_1\bar\sigma_2\bar\sigma_3\lambda} &= 24\delta_{\lambda[\bar\sigma_1}\delta^{\bar\rho}_{\bar\sigma_2}\delta^{\bar\sigma}_{\bar\sigma_3]}, & \tau^{-\bar\rho\bar\sigma}{}_{-\bar\lambda\sigma_1\sigma_2\sigma_3} &= -8\epsilon_{\sigma_1\sigma_2\sigma_3}{}^{[\bar\rho}\delta^{\bar\sigma]}_{\bar\lambda}\,, \\
\tau^{\bar\rho\bar\sigma}{}_{+-\natural\bar\mu_1\bar\mu_2} &= -4\sqrt{2}\delta^{\bar\rho\bar\sigma}_{\bar\mu_1\bar\mu_2}, & \tau^{\bar\rho\bar\sigma}{}_{+-\natural\mu_1\mu_2} &= 2\sqrt{2}\epsilon^{\bar\rho\bar\sigma}{}_{\mu_1\mu_2}\,, \\
\tau^{\bar\rho\bar\sigma}{}_{\natural\bar\sigma_1\bar\sigma_2\bar\sigma_3\lambda} &= 12\sqrt{2}\delta_{\lambda[\bar\sigma_1}\delta^{\bar\rho}_{\bar\sigma_2}\delta^{\bar\sigma}_{\bar\sigma_3]}, & \tau^{\bar\rho\bar\sigma}{}_{\natural\bar\lambda\sigma_1\sigma_2\sigma_3} &= -4\sqrt{2}\epsilon_{\sigma_1\sigma_2\sigma_3}{}^{[\bar\rho}\delta^{\bar\sigma]}_{\bar\lambda}\,.
\end{aligned}
\tag{A.11}
$$

The components of $\tau^{\bar\rho}$ and $\tau^{-\bar\rho}$ are obtained from the above expressions by complex conjugation.

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